THE DYNAMICS OF THE VENTRICULAR WALL AND SOME OBSERVATIONS ON BLOOD FLOW

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ABSTRACT An expression for the energy of motion of the wall of the left ventricle is developed. A cylindrical model is assumed for the left ventricle, and symmetry is used to reduce the problem to a two-dimensional problem. Results obtained indicate that consideration of the energy of motion can be useful in problems of clinical diagnosis. Some correlation between previously published experimental results is also made with the equations derived in this paper.

INTRODUCTION

The relation between wall tension and pressure, as well as the force-velocity relation, have usually been the criteria used to study the performance of the left ventricle. More recently, Dumesnil et al. (3) have stressed the importance of the study of the regional ventricular wall dynamics to assess more adequately the location and extent of dysfunction in ventricles with myocardial disease. Rather than wall thickening, they pointed out that rates of wall thickening give a more distinct differentiation between patients with and without left ventricular disease. In an attempt to quantify these ideas, Gould et al. (4) have examined the motion of the left ventricle wall with a consideration of the power developed during thickening as well as in the circumferential and meridional directions. In so doing, however, they used the expression of the ventricular wall stress as determined with a static model, as well as the velocity of thickening and shortening in the respective directions. This amounts to saying that the work done by a system of forces in equilibrium is always nil, and then expressing the power component due to pressure as a function of the three others (wall thickening, circumferential and meridional power). A more correct approach is to apply the physical principle that the work done by a system of forces is equal to the change in energy (mechanical in this case) of the system. This evidently implies the use of Newton's second law of motion and the study of the inertia forces due to the acceleration of the ventricular wall.

The problem of the inertia forces has already been treated by some authors (10). It was found that the inertia forces amount to 1% of the pressure force and as such can be neglected. We suggest that the study of acceleration and the kinetic and potential energies of motion of the ventricular wall can give a good quantitative formulation of the performance of the left ventricle, and as such can offer an important tool to solve problems involved in clinical diagnosis. Consequently, the starting point of this study is the application of Newton's second law to describe the motion of the ventricular wall and then to derive the expressions of the wall stress and the energy of motion of the system when the inertia forces are taken into consideration.

MATHEMATICAL MODEL

To illustrate our ideas in a simple manner and to avoid complication with the mathematical formulation of the problem, we have used the model for the left ventricle given in the appendix of Gould et al. (4). We consider the left ventricle as a thick-walled cylinder of constant length, contracting radially and symmetrically. The cylinder is made of a uniform and homogeneous material. This is a planar problem, and the application of Newton's second law to an element of volume gives directly (11; see Fig. 1).

$$(\mathrm{d}/\mathrm{d}r)(r\sigma_r) - \sigma_\theta = +\rho r(\mathrm{d}^2 r/\mathrm{d}t^2). \tag{1}$$

We use the relations

$$V_r = \pi r^2 l - \pi a^2 l = \text{constant}, \tag{2}$$

$$(\mathrm{d}r/\mathrm{d}t) = (a/r)(\mathrm{d}a/\mathrm{d}t), \tag{3}$$

$$\rho(\mathrm{d}^2 r/\mathrm{d}t^2) = -\frac{\rho}{r^3}\left(a\frac{\mathrm{d}a}{\mathrm{d}t}\right)^2 + \frac{\rho}{r}\frac{\mathrm{d}}{\mathrm{d}t}\left(a\frac{\mathrm{d}a}{\mathrm{d}t}\right) = -\frac{\alpha}{r^3} + \frac{\beta}{r}. \tag{4}$$

where σ_r is the stress in the radial direction; σ_θ is the stress in the circumferential direction; ρ is the density of the material of the wall; r is the distance measured from the center of the cylinder; a is the inner radius of the cylinder; b is the outer radius of the cylinder; V_r is the volume enclosed between the radius r and the inner radius a.

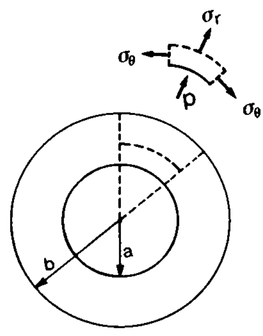

FIGURE 1 Transverse section of a cylinder, showing the internal radius a, the external radius b, the pressure p, and the stresses σ_r and σ_θ.

Substituting from Eq. 4 into Eq. 1 and following the method given by Timoshenko and Goodier (11), one easily obtains.

$$\sigma_r = \left\{\frac{pa^2}{b^2 - a^2} + \frac{\beta}{2(1-\nu)}\frac{a^2 \ln a}{b^2 - a^2} - \frac{\alpha}{2}\left(\frac{1-2\nu}{1-\nu}\right)\frac{\ln a}{b^2 - a^2}\right\}\left\{1 - \frac{b^2}{r^2}\right\}$$

$$+ \left\{\frac{-p_e b^2}{b^2 - a^2} - \frac{\beta}{2(1-\nu)}\frac{b^2 \ln b}{b^2 - a^2} + \frac{\alpha}{2}\left(\frac{1-2\nu}{1-\nu}\right)\frac{\ln b}{b^2 - a^2}\right\}\left\{1 - \frac{a^2}{r^2}\right\}$$

$$- \frac{\alpha}{2}\left(\frac{1-2\nu}{1-\nu}\right)\frac{\ln r}{r^2} + \frac{\beta}{2(1-\nu)}\ln r \tag{5a}$$

$$\sigma_\theta = \left\{\frac{pa^2}{b^2 - a^2} + \frac{\beta}{2(1-\nu)}\frac{a^2 \ln a}{b^2 - a^2} - \frac{\alpha}{2}\left(\frac{1-2\nu}{1-\nu}\right)\frac{\ln a}{b^2 - a^2}\right\}\left\{1 + \frac{b^2}{r^2}\right\}$$

$$+ \left\{\frac{-p_e b^2}{b^2 - a^2} - \frac{\beta}{2(1-\nu)}\frac{b^2 \ln b}{b^2 - a^2} + \frac{\alpha}{2}\left(\frac{1-2\nu}{1-\nu}\right)\frac{\ln b}{b^2 - a^2}\right\}\left\{1 + \frac{a^2}{r^2}\right\}$$

$$- \frac{\alpha}{2}\left(\frac{1-2\nu}{1-\nu}\right)(1 - \ln r) + \frac{\beta}{2(1-\nu)}(1 + \ln r) + \frac{\alpha}{r^2} - \beta \tag{5b}$$

where p_e represents the pressure supposed to act on the external surface and radially inward.

The radial power P_r and the circumferential power P_θ can be readily calculated to be:

$$P_r = -2\pi l a \frac{\mathrm{d}a}{\mathrm{d}t}\left\{\left(\frac{pa^2}{b^2 - a^2} + \frac{\beta}{2(1-\nu)}\frac{a^2 \ln a}{b^2 - a^2} - \frac{\alpha}{2}\left(\frac{1-2\nu}{1-\nu}\right)\frac{\ln a}{b^2 - a^2}\right)\right.$$

$$\cdot\left(\ln\frac{b}{a} - \frac{1}{2}\frac{b^2 - a^2}{a^2}\right)$$

$$+ \left(\frac{-p_e b^2}{b^2 - a^2} - \frac{\beta}{2(1-\nu)}\frac{b^2 \ln b}{b^2 - a^2} + \frac{\alpha}{2}\left(\frac{1-2\nu}{1-\nu}\right)\frac{\ln b}{b^2 - a^2}\right)\left(\ln\frac{b}{a} - \frac{1}{2}\frac{b^2 - a^2}{b^2}\right)$$

$$- \frac{\alpha}{2}\left(\frac{1-2\nu}{1-\nu}\right)\left(\frac{1}{4a^2} - \frac{1}{4b^2} + \frac{1}{2}\frac{\ln a}{a^2} - \frac{1}{2}\frac{\ln b}{b^2}\right)$$

$$\left. + \frac{\beta}{2(1-\nu)}\left(\frac{1}{2}(\ln b)^2 - \frac{1}{2}(\ln a)^2\right)\right\}. \tag{6a}$$

$$P_\theta = 2\pi l a \frac{da}{dt}\left\{\left(\frac{pa^2}{b^2 - a^2} + \frac{\beta}{2(1-\nu)}\frac{a^2 \ln a}{b^2 - a^2} - \frac{\alpha}{2}\left(\frac{1-2\nu}{1-\nu}\right)\frac{\ln a}{b^2 - a^2}\right)\right.$$

$$\cdot\left(\ln\frac{b}{a} + \frac{1}{2}\frac{b^2 - a^2}{a^2}\right)$$

$$+\left(\frac{-p_e b^2}{b^2 - a^2} - \frac{\beta}{2(1-\nu)}\frac{b^2 \ln b}{b^2 - a^2} + \frac{\alpha}{2}\left(\frac{1-2\nu}{1-\nu}\right)\frac{\ln b}{b^2 - a^2}\right)\left(\ln\frac{b}{a} + \frac{1}{2}\frac{b^2 - a^2}{b^2}\right)$$

$$-\frac{\alpha}{2}\left(\frac{1-2\nu}{1-\nu}\right)\left(\frac{1}{4a^2} - \frac{1}{4b^2} + \frac{1}{2}\frac{\ln b}{b^2} - \frac{1}{2}\frac{\ln a}{a^2}\right)$$

$$\left. + \frac{\beta}{2(1-\nu)}\left(\ln\frac{b}{a} + \frac{1}{2}(\ln b)^2 - \frac{1}{2}(\ln a)^2\right) + \frac{\alpha}{2}\left(\frac{1}{a^2} - \frac{1}{b^2}\right) - \beta \ln\frac{b}{a}\right\}. \quad (6b)$$

The total rate of doing mechanical work by the ventricular wall is then given by:

$$P_\theta + P_r = 2\pi l a \frac{da}{dt}\left\{p - p_e + \alpha\left(\frac{1}{2a^2} - \frac{1}{2b^2}\right) - \beta \ln\frac{b}{a}\right\}. \quad (7)$$

The pump power is $2\pi lpa(da/dt)$, and the rate of change of the kinetic energy of motion is given by:

$$\frac{dW}{dt} = 2\pi l a \frac{da}{dt}\left\{\alpha\left(\frac{1}{2a^2} - \frac{1}{2b^2}\right) - \beta \ln\frac{b}{a}\right\}. \quad (8)$$

p_e is approximately zero, and da/dt is negative during contraction, which means that power is actually absorbed by the mechanical system during systole. Eqs. 7 and 8 are valid during systolic motion and during diastolic motion.

It is of interest to note that $\alpha = \rho(a da/dt)^2 = (\rho/4\pi^2 l^2)Q^2$ and $\beta = \rho(d/dt)(a(da/dt))$ can be expressed respectively in terms of the flow Q (cubic centimers per second) and the flow acceleration dQ/dt, in which case Eq. 8 can be rewritten for the ejection phase in the form:

$$\left|\frac{dW}{dt}\right| = +\ Q\left\{\frac{\rho Q^2}{4\pi^2 l^2}\left(\frac{1}{2a^2} - \frac{1}{2b^2}\right) + \rho\frac{dQ}{dt}\frac{1}{2\pi l}\ln\frac{b}{a}\right\}, \quad (9)$$

where we have made the substitution $Q = -dV/dt$, $V = \pi a^2 l$ is the volume of the ventricular chamber. The radial velocity and acceleration are respectively given by:

$$\frac{da}{dt} = -\frac{Q}{2\pi a l} \quad (10a)$$

$$\frac{d^2 a}{dt^2} = -\frac{Q^2}{4\pi a^3 l^2} - \frac{1}{2\pi a l}\frac{dQ}{dt}. \quad (10b)$$

The analogy between Eqs. 9 and 10*b* is to be noted. Eq. 9 takes better into considera-

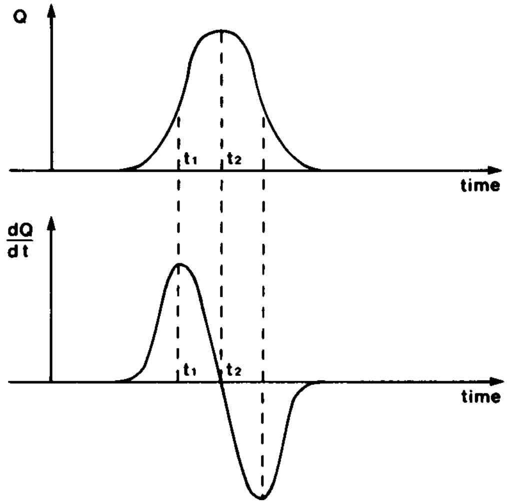

FIGURE 2 Simplified model showing the relative position of the maxima of the flow Q and the flow acceleration dQ/dt with respect to time.

tion the geometry of the cylinder by the inclusion of the terms $(1/2a^2 - 1/2b^2)$ and $\ln(b/a)$. It is to be noted that we can write $Q = Av$, where A is the cross-section area of the aorta supposed constant and v the ejection velocity of blood. Substitution of this expression of Q in Eq. 9 gives an expression similar to that derived by Powell et al. (8) for the energy of blood during ejection of the aorta.

The interesting feature of Eqs. 10*a* and 10*b* is that the time variation of the velocity da/dt and the acceleration d^2a/dt^2 can be readily derived from records of the time variation of the radius a. This latter measurement can be easily performed either by angiographic or echocardiographic methods.

APPLICATION AND DISCUSSION

The potential clinical usefulness of the acceleration is discussed in an instructive article by Noble et al. (7), who noted that the maximum acceleration $(dQ/dt)_{max}$ is by far the most sensitive index of the change in the myocardium. It is important at this point not to confuse the acceleration of the ventricular wall, as expressed by d^2a/dt^2 and the blood acceleration in the aorta given by $dQ/dt = A\,dv/dt$.

At the instant t_2 shown in Fig. 2, one can write:

$$d^2a/dt^2 = -Q_{max}^2/4\pi a^3 l^2, (dQ/dt = 0). \tag{11}$$

A change in the blood flow Q_{max} would result in a relative change in the instantaneous wall acceleration at t_2 given by

$$\frac{(Q_{max} + \Delta Q_{max})^2 - Q_{max}^2}{Q_{max}^2} \approx 2\frac{\Delta Q_{max}}{Q_{max}}. \tag{12}$$

We have reproduced the relative change in the flow acceleration $(dQ/dt)_{max}$ as given in Tables II and III in Noble et al. (7) into column 2 of Table I of the present work.

The results compare fairly well (with one notable exception) with the respective values of $2\Delta Q_{max}/Q_{max}$ given in column 3 of Table I. The results indicate that $\{\Delta(dQ/dt)_{max}/(dQ/dt)_{max}\} \geq 2\Delta Q_{max}/Q_{max}$. Results published by Tallarida et al. (10) indicate that the difference between $\{\Delta(dQ/dt)_{max}/(dQ/dt)_{max}\}$ and $2\Delta Q_{max}/Q_{max}$ increases with the doses of isoproterenol administrated to the dogs under test. This tends to confirm the result of Noble et al. (7) that the acceleration is more sensitive than the flow measurement to changes in the myocardium. Eqs. 10 and 11 indicate that both relative changes of the flow Q_{max} and the flow acceleration $(dQ/dt)_{max}$ are directly induced by similar relative changes in the wall velocity and acceleration, and a study of these latter would be more indicated for clinical application. It is also evident that the quantity dW/dt in Eq. 9, being proportional to Q^3_{max} at the instant t_2 (see Fig. 2) would be more sensitive than the acceleration to changes in myocardial function, its relative variation being of the order of $3\Delta Q_{max}/Q_{max}$.

It is now interesting to compare the relative variation of the peak power reported in Table II of Noble et al. (7), with the results predicted by Eq. 9. It is difficult to know exactly the variation of the term $Q_x(dQ/dt)_{max}$ because the value of Q_x is not given at the instant t_1 shown in Fig. 2. (For the first case of Table II of Noble et al., the relative variation of $Q_x(dQ/dt)_{max}$ is 0.49 and 0.96, depending whether we take, $Q_x = Q_{max}/3$

TABLE I

COMPARISON OF THE RELATIVE VARIATIONS OF THE PEAK FLOW Q, PEAK FLOW ACCELERATION $\frac{dQ}{dt}$ AND PEAK POWER

Exp.	$\frac{\Delta Q_{max}}{Q_{max}}$	$\frac{\Delta\left(\frac{dQ}{dt}\right)_{max}}{\left(\frac{dQ}{dt}\right)_{max}}$	$\frac{\Delta(Q_{max})^2}{Q^2_{max}} \approx 2\frac{\Delta Q_{max}}{Q_{max}}$	$\frac{\Delta(Q_{max})^3}{Q^3_{max}} \approx 3\frac{\Delta Q_{max}}{Q_{max}}$	(Relative variation of the peak power of blood) ×2	(Relative variation of the stroke power of blood) ×2
1	0.22	0.44	0.44	0.66	0.64	0.75
2	0.26	0.61	0.52	0.78	0.84	0.74
3	0.22	0.5	0.44	0.66	0.86	0.68
4	0.18	0.45	0.36	0.54	0.42	0.22
5	0.1	0.4	0.2	0.3	0.3	0.62
6	0.01	0.35	0.02	0.03	0.04	0.02
7*	0.17	0.38	0.34	0.51		
8*	0.08	0.16	0 16	0.24		
9*	0.12	0.24	0.24	0.36		
10*	0.06	0.34	0.12	0.12		

Each row refers to one dog under test. Correlation between the numerical values reported are explained in the text.

Values in columns 1, 2, 5, and 6 are taken from Table II of Noble et al. (7).

*Values in columns 1 and 2 are taken from Table III of Noble et al. (7).

TABLE II

COMPARISON OF THE RATIOS OF PRESSURES WITH Q/Q_{ref} AND $(Q/Q_{ref})^{1/2}$. ALL VALUES OF PRESSURE AND FLOW REFER TO PEAK VALUES. EACH ROW REFERS TO ONE DOG UNDER TEST. THE MAXIMUM FLOW Q_{ref} AND THE MAXIMUM PRESSURE p_{ref} REFER TO THE CONTROL VALUES. NOTE THAT THE DIFFERENCES $|p/p_{ref} - (Q/Q_{ref})^{1/2}|$ ARE NEARER TO ZERO THAN THE DIFFERENCES $|p/p_{ref} - Q/Q_{ref}|$, WITH THE EXCEPTION OF EXPERIMENT 3.

Exp.	Q_{ref}	p_{ref}	Q	p	$\frac{Q}{Q_{ref}}$	$\frac{p}{p_{ref}}$	$\left(\frac{Q}{Q_{ref}}\right)^{1/2}$	$\left\lvert\frac{p}{p_{ref}} - \left(\frac{Q}{Q_{ref}}\right)^{1/2}\right\rvert$	$\left\lvert\frac{p}{p_{ref}} - \frac{Q}{Q_{ref}}\right\rvert$
	ml/s	*mm HG*	*ml/s*	*mm Hg*					
1	306	132	372	142	1.215	1.076	1.102	0.026	0.139
2	306	132	386	147	1.261	1.114	1.123	0.009	0.147
5	460	130	505	137	1.098	1.054	1.048	0.006	0.044
3	294	135	358	158*	1.218	1.17	1.103	0.067	0.048
4	281	150	332	154*	1.181	1.027	1.087	0.06	0.154
6	715	142	725	143*	1.0139	1.007	1.0069	0.0001	0.0069

Columns 1–4 are taken from Table II of Noble et al. (7).
*Peak left ventricular pressure.

or $Q_{max}/2$, respectively). Using the same reasoning as for the acceleration, we note that dW/dt is proportional to Q^3_{max} at the instant t_2 ($dQ/dt = 0$). As already mentioned, one would expect a relative variation of the quantity dW/dt of the order of $3\Delta Q_{max}/Q_{max}$ for the various experimental cases reported in Table II of Noble et al. We have written the quantity $3\Delta Q_{max}/Q_{max}$ in Table I together with the values of the relative change in peak power. (We have multiplied these latter values by 2 to compare with columns 4 and 5 of Table I.) The results are very nearly equal for three out of six cases considered.

Small kinetic terms neglected, the blood energy is given by the product pQ (p in the following will always stand for pressure). The previous result suggests a relation between the arterial pressure p and the flow Q of the form $p \propto Q^{1/2}$. Inasmuch as the maximum pressure in the aorta and the maximum flow occur simultaneously (9), it is possible to compare p_{max} and Q_{max} to verify the relation $p_{max} \propto Q^{1/2}_{max}$. We have done this in Tables II–IV. All the values used for the pressure and flow are maximum values, and consequently we shall omit in the following the subscript "max." We have computed in Table II the values p/p_{ref}, $Q^{1/2}/Q^{1/2}_{ref}$, Q/Q_{ref}, using the values of Table II of Noble et al. The reference values p_{ref} and Q_{ref} refer to the control values. The ratios of columns 5–7 are computed for the same heart, i.e., factors involving the geometry of the aorta cancel out. It is seen that the value of p/p_{ref} is nearer to the value of $Q^{1/2}/Q^{1/2}_{ref}$ than it is to the value of Q/Q_{ref}. To get more evidence for this result, in Tables III and IV we have computed the previous ratios for results given in Table I of Noble et al. (7) and Table I of Spencer et al. (9). The reference values are here taken as the first maximum values of pressure p and flow Q in the respective tables, and it is to be pointed out that we are comparing in this case ratios between values measured for different hearts. One consequently expects the discrepancy between the results to be

TABLE III

COMPARISON OF THE RATIOS p/p_{ref} WITH Q/Q_{ref} AND $Q^{1/2}/Q_{ref}^{1/2}$

Q	p	$\frac{Q}{Q_{ref}}$	$\frac{p}{p_{ref}}$	$\frac{Q^{1/2}}{Q_{ref}^{1/2}}$	$\left\lvert\frac{p}{p_{ref}} - \frac{Q^{1/2}}{Q_{ref}^{1/2}}\right\rvert$	$\left\lvert\frac{p}{p_{ref}} - \frac{Q}{Q_{ref}}\right\rvert$
ml/s	*mm Hg*					
415	115	0.883	0.943	0.939	0.004	0.06
367	113	0.781	0.926	0.884	0.042	0.145
520	114	1.106	1.052	0.934	0.118	0.054
380	121	0.808	0.992	0.899	0.093	0.184
587	135	1.249	1.106	1.117	0.011	0.143
520	154	1.106	1.262	1.052	0.21	0.156
380	143	0.808	1.172	0.899	0.273	0.364
278	158	0.591	1.295	0.769	0.526	0.704
400	470	0.851	0.992	0.922	0.07	0.141
365	115	0.777	0.943	0.881	0.062	0.166
276	146	0.587	1.197	0.766	0.431	0.61
333	132	0.708	1.082	0.842	0.24	0.374
460	115	0.979	0.943	0.989	0.046	0.036
550	110	1.17	1.082	1.08	0.002	0.088

All values of pressure and flow refer to peak values. Each row refers to one dog under test, and the reference values p_{ref} and Q_{ref} refer to maximum pressure and flow of another dog. Note that the differences $|p/p_{ref} - (Q/Q_{ref})^{1/2}|$ are smaller than the differences $|p/p_{ref} - Q/Q_{ref}|$.
Columns 1 and 2 are taken from Table I of Noble et al. (7). p_{ref} = 122 mm Hg; Q_{ref} = 470 ml/s.

higher than in Table II. With three exceptions, the general tendency is still evident, that p/p_{ref} is nearer to the value of $(Q/Q_{ref})^{1/2}$ than it is to value of Q/Q_{ref}.

Another check on the relation $p \propto Q^{1/2}$ has been done by studying the variation (peak power)/maximum flow)$^{3/2}$ with (peak power)/max flow)^2 ($p_{max}Q_{max} \propto Q_{max}^{3/2}$ if $p_{max} \propto Q_{max}^{1/2}$, and $p_{max}Q_{max} \propto Q_{max}^2$ if $p_{max} \propto Q_{max}$). Percentage variation of these

TABLE IV

COMPARISON OF THE RATIOS OF p/p_{ref} WITH Q/Q_{ref} AND $Q^{1/2}/Q_{ref}^{1/2}$

Q	p	$\frac{Q}{Q_{ref}}$	$\frac{p}{p_{ref}}$	$\frac{Q^{1/2}}{Q_{ref}^{1/2}}$	$\left\lvert\frac{p}{p_{ref}} - \frac{Q^{1/2}}{Q_{ref}^{1/2}}\right\rvert$	$\left\lvert\frac{p}{p_{ref}} - \frac{Q}{Q_{ref}}\right\rvert$
ml/s	*mm Hg*					
119.6	95	1.318	0.864	1.148	0.284	0.454
68.75	112	0.757	1.018	0.87	0.148	0.261
199.8	110	2.201	1	1.484	0.484	1.201
140.65	90	1.55	0.818	1.245	0.427	0.732
94.75	78	1.044	0.709	1.022	0.313	0.335
89.6	120	0.987	1.091	0.994	0.097	0.104
143.75	130	1.584	1.182	1.258	0.076	0.402
67.33	107	0.742	0.973	0.861	0.112	0.231

Comparison of the ratios p/p_{ref} with Q/Q_{ref} and $(Q/Q_{ref})^{1/2}$. All values of pressure and flow refer to peak values. Each row refers to one dog under test, and the reference values p_{ref} and Q_{ref} refer, respectively, to maximum pressure and flow of another dog. Note that the differences $|p/p_{ref} - (Q/Q_{ref})^{1/2}|$ are smaller than the differences $|p/p_{ref} - Q/Q_{ref}|$.
Columns 1 and 2 are taken from Table I, ref. 9. Q_{ref} = 90.75 ml/s, p_{ref} = 110 mm Hg.

TABLE V
COMPARISON OF THE RELATIVE VARIATIONS OF (PEAK POWER)/(MAX FLOW)$^{\delta}$ FOR $\delta = 3/2$ AND $\delta = 2$.

Exp.	Peak power	Max. flow	Peak power / $(\text{Max flow})^{3/2}$	$\lvert\Delta\rvert$	Peak power / $(\text{Max. flow})^{2}$	$\lvert\Delta\rvert$
	ergs × 10^6	*ml/s*	× 10^6	%	× 10^6	%
1	52.5	306	0.009808		0.0005607	
	70	372	0.009756	0.53	0.00050584	9.78
	75	386	0.009889	0.83	0.0005034	10.22
3	52	294	0.010315		0.0006016	
	75	358	0.011072	7.34	0.0005852	2.73
4	55	281	0.011676		0.0006965	
	67.5	332	0.011158	4.43	0.0006124	12.09
5	79	460	0.008007		0.0003733	
	91	505	0.00802	0.14	0.0003568	4.42
6	134	715	0.007008		0.0002621	
	137	725	0.007018	0.13	0.0002606	0.56

Absolute variation $\lvert \Delta\% \rvert$ of the quantities (peak power)/(peak flow)$^{3/2}$ and (peak power)/(peak flow)^2. Note that the variation is smaller in the first case than in the second, with the exception of experiment 3. Columns 1 and 2 are taken from Table II of Noble et al. (7). First row of each experiment corresponds to the control, second or third row corresponds to the effect of intracoronary injections of calcium or isopropylnoradrenaline.

quantities with respect to respective control values is reported in columns 4 and 6 of Table V. With one exception, it is seen that the variation of the first quantity is smaller than that of the second quantity, thus confirming the result that the relation $p_{max} \propto Q_{max}^{1/2}$ describes more correctly the actual behavior of blood in the aorta. There is only one exception in Tables I, II, and V, corresponding to the same experiment (number 3) of Noble's Table II (7), that suggests that $p_{max} \propto Q_{max}$, and one may wonder if a relation of the form $p_{max} \propto Q_{max}^{\alpha}$ would not be generally more correct. (See Appendix A.)

Recently, Nichols et al. (6) have published results obtained from human subjects that confirm the previous result that $p_{max} \propto Q_{max}^{\alpha}$. This evidently put some reserves about the assumption of linearity between the pressure and the flow that these authors used in conjunction with the Fourier method to calculate the external hydraulic power of the left ventricle. More important is the error made in the work of Nichols et al. (6) as well as in the work of Milnor et al. (5) by neglecting the term proportional to $Q(\mathrm{d}Q/\mathrm{d}t)$ appearing in the expression of the power of the ventricular wall (Eq. 9), and which for blood power has been correctly accounted for in the work of Powell et al. (8).

This term arises simply from the fact that the rate of change of kinetic energy K for a mass m in motion with velocity u is given by

$$\mathrm{d}K/\mathrm{d}t = \tfrac{1}{2}(\mathrm{d}m/\mathrm{d}t)u^2 + mu(\mathrm{d}u\mathrm{d}t). \quad (13)$$

The first term of Eq. 13 corresponds to the term Q^3, whereas the second term corresponds to the term $Q\mathrm{d}Q/\mathrm{d}t$.

To have an idea in the relative maxima of these two quantities as they appear in Eq. 9, we have assumed the inner radius $a = 2.1$ cm, the outer radius $b = 2.9$ cm, length $l = 5$ cm for standard dogs. With the results of Noble et al. (7), we find that the maximum value of the first term in Eq. 9 is of the order of 5.98×10^3 erg/s, whereas the second term calculated at max. dQ/dt (the first term being neglected) is of the order of 28×10^3 erg/s. Hence the two terms have maxima of the same order of magnitude.

We finally note that an interesting quantity to investigate is

$$\frac{Q_{max}^2}{V^2} = -4\left(\frac{1}{a}\frac{d^2a}{dt^2}\right). \tag{14}$$

The negative sign in Eq. 14 means that during contraction, the acceleration is directed contrary to the positive radial direction. Eq. 14 is an expression for the normalized acceleration of the cylindrical wall at the instant the flow Q is maximal. Unfortunately, we have been unable to find simultaneous values of the ventricular chamber volume V and Q_{max} to study the variation of Eq. 14 for different clinical cases.

CONCLUSION

Previous experimental results (6–9) and the present work suggest that the study of the dynamics of the ventricular wall can lead to interesting applications in various fields of clinical interest. It is unfortunate that most of the experimental results published to date are confined to static models, peak values, or average values, whereas exact time-to-time variations of the different parameters describing the ventricular function are necessary for a full understanding of the performance of the left ventricle. This is true in particular for the study of the relation between the mass of the ventricle (or the parameters describing the geometry of the ventricular wall) and its pump function (or the parameters describing blood ejection). It is to be noted for instance that we have been able to correlate only between relative variations of the different quantities involved, because then the values of the inner radius a, outer radius b, and length l of the ventricle cancel out.

Finally, in establishing the relation $p_{max} \propto Q_{max}^{1/2}$, we have used three criteria: (*a*) comparing the relative variation involving the quantities Q_{max}^3 with the relative variation of peak powers (Table I); (*b*) comparing the ratio of pressures respectively with the ratio of the quantities $Q_{max}^{1/2}$ and Q_{max} (Tables II–IV); (*c*) and comparing the ratio of peak powers respectively with the ratio of the quantities $Q_{max}^{3/2}$ and Q_{max}^2 (Table V). Further discussion of the problem of nonlinearity is given in the references mentioned in the work of Nichols et al. (6).

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APPENDIX A

The linear model for blood circulation was usually assumed on the basis of studies whereby the heart rate is varied, with the consequent variation of magnitude, phase, and frequency of the pressure harmonics. The only changes obtained were within the limits of experimental error (1). To see the inherent difficulty of this method, consider the following equation:

$$(p + \Delta p)/(Q + \Delta Q) \approx (p/Q)(1 + \Delta p/p - \Delta Q/Q). \qquad \text{(A1)}$$

If we write:

$$p/Q = Z \qquad \text{(A2)}$$

where Z is the input impedance, then

$$\Delta p/p - \Delta Q/Q \approx \Delta Z/Z. \qquad \text{(A3)}$$

In the case where $p \propto Q^{1/2}$, then $\Delta p/p \propto \frac{1}{2}\Delta Q/Q$. We have reproduced the values of $\Delta p/p$, $\Delta Q/Q$, $\frac{1}{2}\Delta Q/Q$, and $(\Delta p/p)/(\Delta Q/Q)$ in Table VI, and it is evident that $(\Delta p/p)/(\Delta Q/Q)$ is far from being equal to 1 (except for experiment 3). From Table II of Bergel et al. (1), we have $\Delta Z_p/Z_p = 0.1/3.36 = 0.029$ for pulsatile flow, of the same order of magnitude as the values of $(\Delta p/p - \Delta Q/Q)$ of experiments 4–6 in Table VI, and consequently this relative variation in $\Delta Z_p/Z_p$ cannot be considered negligible. In the work of Dick et al. (2), it was observed that the sum frequency terms in the harmonics generated were more pronounced in flows than in pressures, which fact does suggest a relation of the form $Q \propto p^2$, or inversely $p \propto Q^{1/2}$. These authors report a variation of the order of 2% in the power of the abdominal aortic flow at the sum frequency terms of the harmonics generated and as such they find that the assumption of near linearity is justified for most purposes. But it is to be noted that $\Delta Z/Z$ in Table VI for experiments 5 and 6 varies from 0.7 to 5%, which does not mean at all that $p_{max} \propto Q_{max}$. On the contrary, experiments 5 and 6 of Table VI strongly suggest that $p_{max} \propto Q_{max}^{1/2}$. It is unfortunate that the values of pressure p and flow Q have not been reported

TABLE VI

COMPARISON OF THE RELATIVE VARIATION OF THE PRESSURE WITH THE RELATIVE VARIATION OF THE FLOW

Exp.	$Q_{ref.}$	p_{ref}	Q	p	$\frac{\Delta Q}{Q}$	$\frac{\Delta p}{p}$	$\frac{1}{2}\frac{\Delta Q}{Q}$	$\alpha \approx \left(\frac{\Delta p}{p} / \frac{\Delta Q}{Q}\right)$
	ml/s	*mm Hg*	*ml/s*	*mm Hg*				
1	306	132	372	142	0.216	0.076	0.108	0.352
2	306	132	386	147	0.261	0.114	0.131	0.437
3	294	135	358	158	0.218	0.170	0.109	0.78
4	281	150	332	154	0.182	0.027	0.091	0.148
5	460	130	505	137	0.098	0.054	0.049	0.55
6	715	142	725	143	0.014	0.007	0.007	0.5

Relative variations of flow $\Delta Q_{max}/Q_{max}$, pressure $\Delta p_{max}/p_{max}$ and $1/2\,\Delta Q_{max}/Q_{max}$. Note that $\Delta p_{max}/p_{max}$ is nearer to the value of $1/2\,\Delta Q_{max}/Q_{max}$ than $\Delta Q_{max}/Q_{max}$, with the exception of experiment 3. The values of α in the relation $p_{max} \propto Q_{max}^{\alpha}$ is indicated in the last column. The flow $Q_{ref} \equiv Q_{max}$ and the pressure $p_{ref} \equiv p_{max}$ refer to control values.
Values of columns 2–5 are taken from Table II of Noble et al. (7).

in the work of Dick et al. (2). The above discussion suggests that a small relative variation can be more significant than it would first appear.

APPENDIX B

In the preceding discussion, we have not considered the effect of the contracting force during systolic motion. Although this does not affect the expression of the rate of change of kinetic energy as given by Eqs. 8 and 9, we want in the following to indicate what would be the effect of considering contracting forces.

We represent in the following the contracting force as a body force of density B per unit mass acting symmetrically in the radial direction. Eq. 1 becomes:

$$(\mathrm{d}/\mathrm{d}r)(r\sigma_r) - \sigma_\theta - \rho B r = \rho r(\mathrm{d}^2 r/\mathrm{d}t^2). \quad \text{(B1)}$$

Solution of Eq. B1 by the method developed by Timoshenko and Goodier (11) for a two-dimensional model gives:

$$t_r = \sigma_r + \frac{1}{3}\frac{2-\nu}{1-\nu}\frac{Da^3}{b^2-a^2}\left(1-\frac{b^2}{r^2}\right) - \frac{1}{3}\frac{2-\nu}{1-\nu}\frac{Db^3}{b^2-a^2}\left(1-\frac{a^2}{r^2}\right) + \frac{1}{3}\frac{2-\nu}{1-\nu}Dr \quad \text{(B2)}$$

$$t_\theta = \sigma_\theta + \frac{1}{3}\frac{2-\nu}{1-\nu}\frac{Da^3}{b^2-a^2}\left(1+\frac{b^2}{r^2}\right) - \frac{1}{3}\frac{2-\nu}{1-\nu}\frac{Db^3}{b^2-a^2}\left(1+\frac{a^2}{r^2}\right) + \frac{1}{3}\frac{1-\nu}{1-\nu}Dr \quad \text{(B3)}$$

σ_r and σ_θ are given by eqs. 5*a* and 5*b*, $D = \rho B$ and t_r, t_θ are the stresses in the radial and tangential directions, respectively, in the presence of body forces of density B per unit mass.

The expression of the power in the radial and tangential directions are given by:

$$P_r' = P_r - 2\pi l a \frac{\mathrm{d}a}{\mathrm{d}t}\left\{\frac{1}{3}\frac{2-\nu}{1-\nu}\frac{Da^3}{b^2-a^2}\left[\ln\frac{b}{a} - b^2\left(\frac{1}{2a^2}-\frac{1}{2b^2}\right)\right] - \frac{1}{3}\frac{2-\nu}{1-\nu}\frac{Db^3}{b^2-a^2}\left[\ln\frac{b}{a} - a^2\left(\frac{1}{2a^2}-\frac{1}{2b^2}\right)\right] + \frac{1}{3}\frac{2-\nu}{1-\nu}D(b-a)\right\} \quad \text{(B4)}$$

$$P_\theta' = P_\theta + 2\pi l a \frac{\mathrm{d}a}{\mathrm{d}t}\left\{\frac{1}{3}\frac{2-\nu}{1-\nu}\frac{Da^3}{b^2-a^2}\left[\ln\frac{b}{a} + b^2\left(\frac{1}{2a^2}-\frac{1}{2b^2}\right)\right] - \frac{1}{3}\frac{2-\nu}{1-\nu}\frac{Db^3}{b^2-a^2}\left[\ln\frac{b}{a} + a^2\left(\frac{1}{2a^2}-\frac{1}{2b^2}\right)\right] + \frac{1}{3}\frac{1+\nu}{1-\nu}D(b-a)\right\} \quad \text{(B5)}$$

P_r and P_θ are given by Eqs. 6*a* and 6*b*. The total power in presence of constant body forces of density D per unit volume is then given by:

$$P'_r + P'_\theta = 2\pi la \frac{\mathrm{d}a}{\mathrm{d}t}\left\{-D(b-a) + p - p_e + \alpha\left(\frac{1}{2a^2} - \frac{1}{2b^2}\right) - \beta \ln \frac{b}{a}\right\}. \quad \text{(B6)}$$

p_e, the pressure on the external wall, is usually taken as equal to zero. Since during contraction $\mathrm{d}a/\mathrm{d}t$ is negative, Eq. B6 shows that the generated power $\{-D(b-a) \times 2\pi la(\mathrm{d}a/\mathrm{d}t)\}$ is positive, while the absorbed mechanical power is negative. If we assume total balance of power, other forms of energy being neglected, we can write that power generated equals mechanical power absorbed or:

$$2\pi la \frac{\mathrm{d}a}{\mathrm{d}t} D(b-a) = 2\pi la \frac{\mathrm{d}a}{\mathrm{d}t}\left\{p - p_e + \alpha\left(\frac{1}{2a^2} - \frac{1}{2b^2}\right) - \beta \ln \frac{b}{a}\right\}. \quad \text{(B7)}$$

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